DYNAMIC PARAMETERS OF DC ELECTRIC MOTOR DRIVES
Control System Elements course

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1 Task description

The goal of the exercise was to get acquainted with graph-analytical methods for determining the dynamic parameters of DC electric motor drives.

2 Tasks for preparation and testing the knowledge

2.1

To determine the constants a and b, so that n of the calculated points are as close as possible to the direction and that the root mean square error is minimal, the method should be as follows:

- choose some Δt,
- for different values of t_1 calculate I_{a0} for t_1, I_{a1} for $\Delta t + t_1$, I_{a2} for $2\Delta t + t_1$ and I_{a3} for $3\Delta t + t_1$ (or respectively Ω_0, Ω_1, Ω_2 and Ω_3),
- store points $\frac{I_{a0}-I_{a3}}{I_{a0}-I_{a1}}$ and $\frac{I_{a0}-I_{a2}}{I_{a0}-I_{a1}}$ (respectively $\frac{\Omega_0-\Omega_3}{\Omega_0-\Omega_1}$ $\frac{\Omega_0-\Omega_2}{\Omega_0-\Omega_1}$),
- apply linear regression on stored points so that a line y=ax+b is the best approximation in the mean square error criterion.

From the appendix in the lecture we can see that the following equation is true:

$$\frac{I_{a3}}{I_{a1}} = (e^{s_1 \Delta t} + e^{s_2 \Delta t})\frac{I_{a2}}{I_{a1}} - e^{-\frac{\Delta t}{T_a}}, \tag{1}$$

so the calculated points are scattered around the direction of some function y=ax+b (similarly for Ω), in our case derived in the laboratory introduction.

2.2 Analysis of the DC motor model

Using Simulink tool in Matlab, I have simulated the transient characteristics for the DC motor model in the task. I have presented the general shapes of the characteristics on the figure 1.

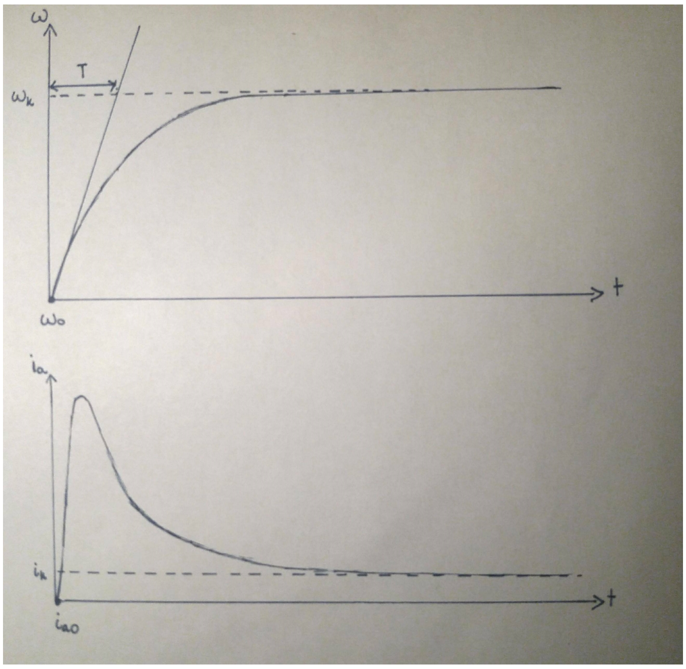

Figure 1: Transient characteristics.

However, three cases have to be analyzed:

- B $= 0$, $M_t \neq 0$,
- B $\neq 0$, $M_t = 0$,
- B $= 0$, $M_t = 0$.

The element that changes in these three cases are steady states, initial values and the time constant if it exists. It can be derived that:

$$\omega_0 = \frac{K_a K}{K_a K^2 + B} U_{a0} - \frac{1}{K_a K^2 + B} M_{t0}, \tag{2}$$

$$i_{a0} = \frac{B}{K^2} U_{a0} + \frac{K_a K}{K_a K^2 + B} M_{t0}, \tag{3}$$

$$T = \frac{J}{K_a K^2 + B}, \tag{4}$$

$$\omega_k = \omega_0 + \frac{K_a K}{K_a K^2 + B} U_a, \tag{5}$$

$$i_k = i_{a0} + \frac{B}{K^2} U_a. \tag{6}$$

If B $= 0$, $M_t \neq 0$:

$$\omega_0 = \frac{1}{K} U_{a0} - \frac{1}{K_a K^2} M_{t0}, \tag{7}$$

$$i_{a0} = \frac{1}{K} M_{t0}, \tag{8}$$

$$T = \frac{J}{K_a K^2}, \tag{9}$$

$$\omega_k = \omega_0 + \frac{1}{K} U_a, \tag{10}$$

$$i_k = i_{a0}. \tag{11}$$

If B $\neq 0$, $M_t = 0$:

$$\omega_0 = \frac{K_a K}{K_a K^2 + B} U_{a0}, \tag{12}$$

$$i_{a0} = \frac{B}{K^2} U_{a0}, \tag{13}$$

$$T = \frac{J}{K_a K^2 + B}, \tag{14}$$

$$\omega_k = \omega_0 + \frac{K_a K}{K_a K^2 + B} U_a, \tag{15}$$

$$i_k = i_{a0} + \frac{B}{K^2} U_a. \tag{16}$$

If B=0 and $M_t = 0$:

$$\omega_0 = \frac{1}{K} U_{a0}, \tag{17}$$

$$i_{a0} = 0, \tag{18}$$

$$T = \frac{J}{K_a K^2}, \tag{19}$$

$$\omega_k = \omega_0 + \frac{1}{K} U_a, \tag{20}$$

$$i_k = i_{a0} = 0. \tag{21}$$

2.3 Analysis of the DC motor model II

Using Simulink tool in Matlab, I have simulated the transient characteristics for the DC motor model in the task. I have presented the general shapes of the characteristics on the figure 2. The assumption is that $L_{au} \neq 0$. The engine torque increases to the amount needed to start the engine when the current results in torque that is greater than M_t. It can be derived that:

$$\omega_0 = \frac{1}{K} U_{a0} - \frac{1}{KaK^2} M_{t0} \tag{22}$$

$$i_{a0} = \frac{1}{K} M_{t0} \tag{23}$$

$$\omega_k = \omega_0 + \frac{1}{K} U_a \tag{24}$$

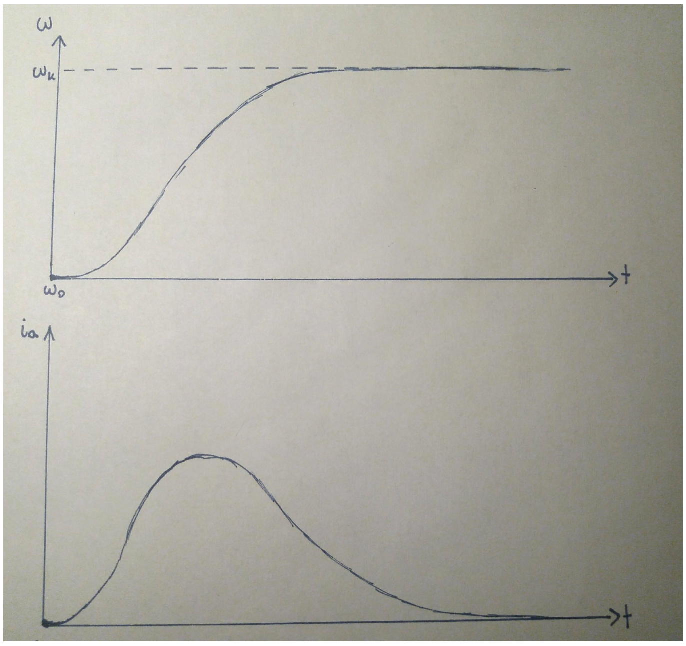

Figure 2: Transient characteristics for $L_{au} \neq 0$.

3 Work on the exercise

3.1 Determining T_a and T_m of DC motors

3.1.1 DC motor No 1

To make the further operations simpler, the response of the system has been smoothed with the use of polyfit function. The following approximation was obtained:

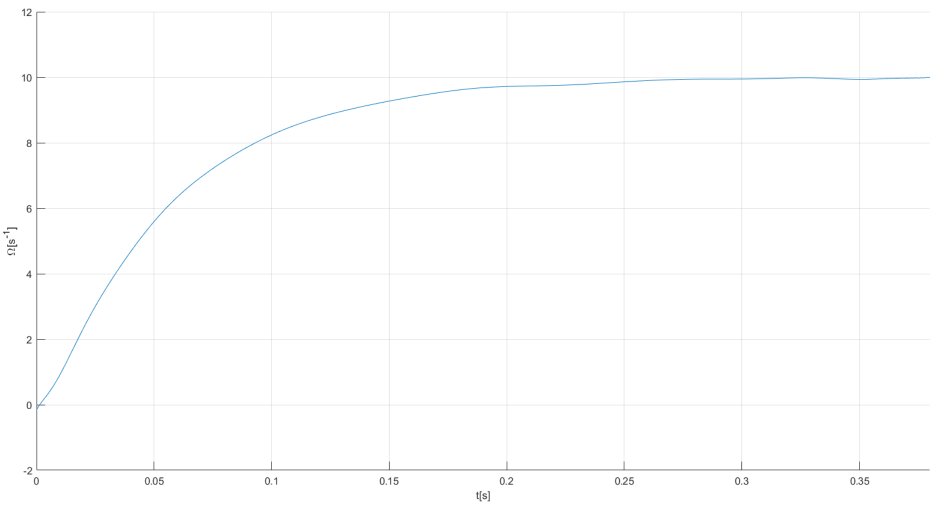

Figure 3: Smoothed input transient.

To calculate the electromechanical time constant, the area between the transient and the line describing steady state value must be computed first. With the use of matlab tools:

$$T_m = \frac{P}{\Omega_0} \approx \frac{0.6166}{10.0} \approx 0.06166[s]. \tag{25}$$

With the value $\Delta t = 0.05$ms, Ω_0 for t_1, Ω_1 for $t_1 + \Delta t$, Ω_2 for $t_1 + 2\Delta t$ and Ω_3 for $t_1 + 3\Delta t$ were calculated for different choices of t_1 and the equivalent points $\frac{\Omega_0 - \Omega_2}{\Omega_0 - \Omega_1}$ and $\frac{\Omega_0 - \Omega_3}{\Omega_0 - \Omega_1}$ were plotted. All these points got approximated by the polynomial of order 1 (y=ax+b), where a=1.8562, b=-0.8905.

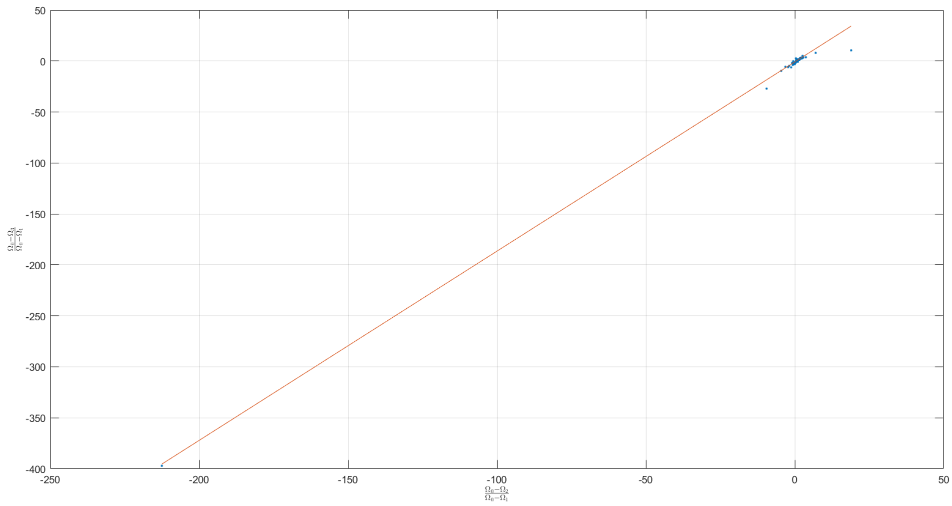

Figure 4: Linear approximation for the plotted points.

From the obtained coefficients of this approximation the reinforcement time constant can be calculated:

$$T_a = \frac{\Delta t}{ln(-b)} \approx 0.0086[s]. \tag{26}$$

While comparing the model with the data, I saw that the model is slightly shifted, so I changed $T_a = 0.0086$ to $T_a = 0.0046$ and the model matched the data much better.

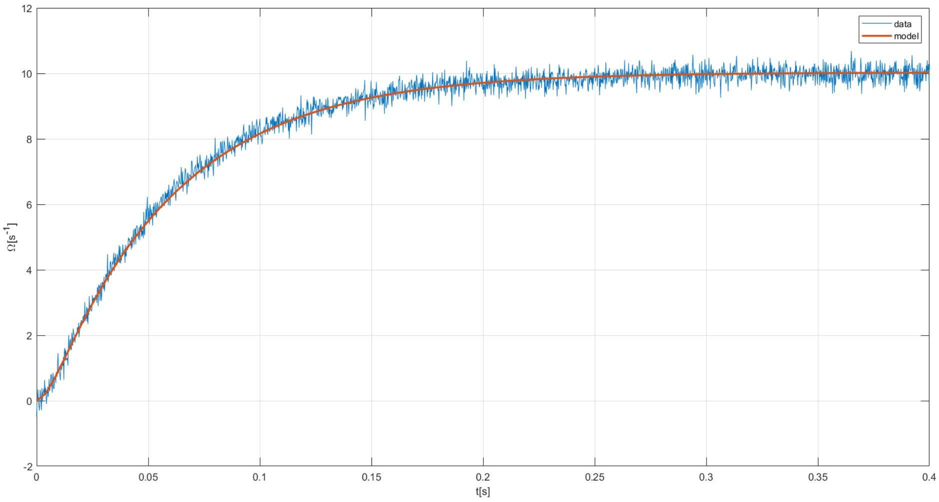

Figure 5: Model comparison with the data points.

So finally, the parameters are as follows:

- $T_a = 0.0046[s]$,
- $T_m = 0.06166[s]$.

3.1.2 DC motor No 2

In this case we follow the same procedure as in the previous case. To make the further operations simpler, the response of the system has been smoothed with the use of polyfit function. The following approximation was obtained:

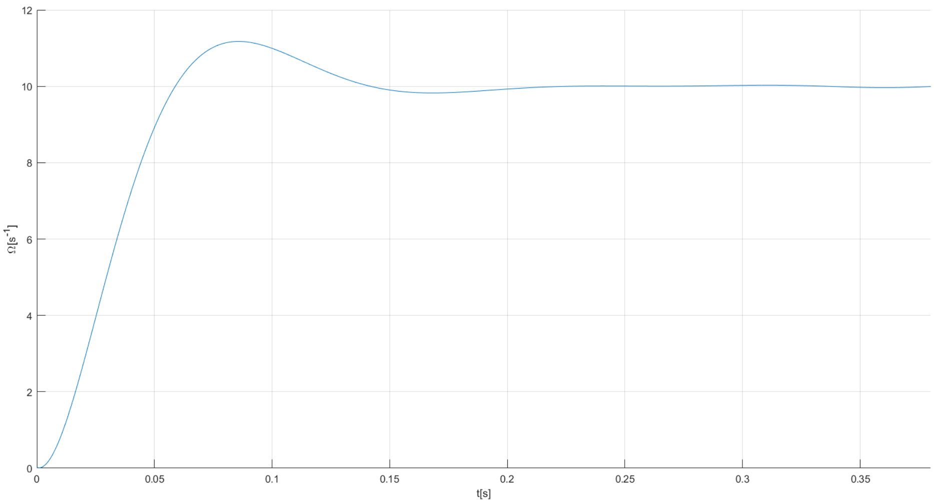

Figure 6: Smoothed input transient.

To calculate the electromechanical time constant, the area between the transient and the line describing steady state value must be computed first. With the use of matlab tools:

$$T_m = \frac{P}{\Omega_0} \approx \frac{0.2702}{10.0} \approx 0.02702[s]. \tag{27}$$

With the value $\Delta t = 0.05$ms, Ω_0 for t_1, Ω_1 for $t_1 + \Delta t$, Ω_2 for $t_1 + 2\Delta t$ and Ω_3 for $t_1 + 3\Delta t$ were calculated for different choices of t_1 and the equivalent points $\frac{\Omega_0 - \Omega_2}{\Omega_0 - \Omega_1}$ and $\frac{\Omega_0 - \Omega_3}{\Omega_0 - \Omega_1}$ were plotted. All these points got approximated by the polynomial of order 1 (y=ax+b), where a=1.9543, b=-0.9524.

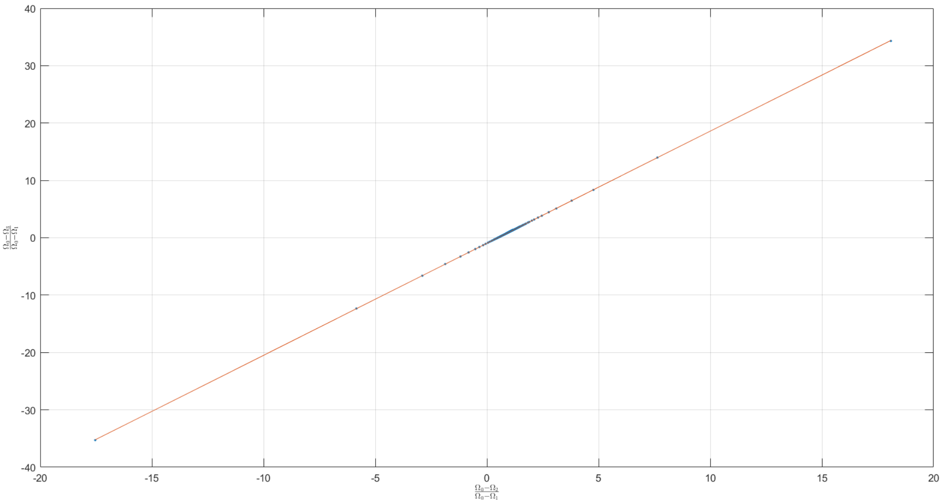

Figure 7: Linear approximation for the plotted points.

From the obtained coefficients of this approximation the reinforcement time constant can be calculated:

$$T_a = \frac{\Delta t}{ln(-b)} \approx 0.0205[s]. \tag{28}$$

Below, the comparison of the model and the input data is presented.

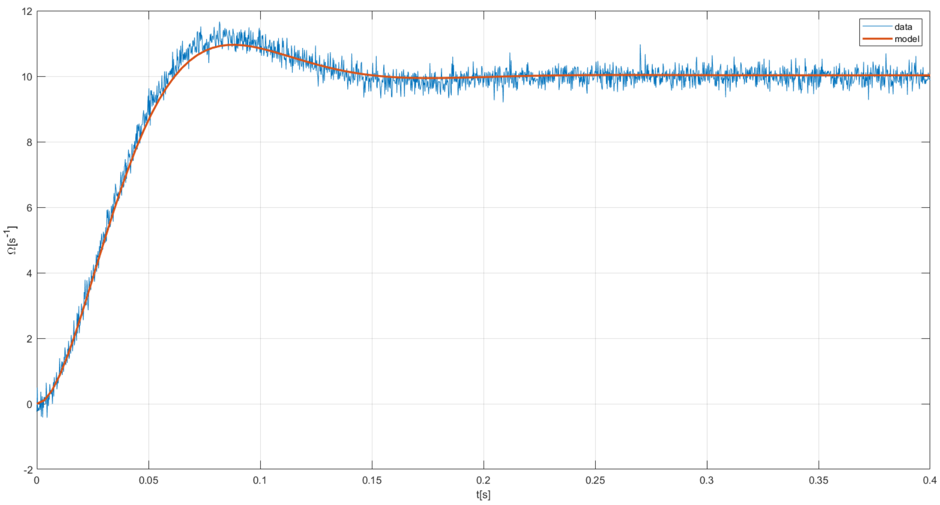

Figure 8: Model comparison with the data points.

So finally, the parameters are as follows:

- $T_a = 0.0205[s]$,
- $T_m = 0.02702[s]$.

3.1.3 DC motor No 3

The method used in this case differs from the one presented above, but the start is similar - polynomial approximation is applied to the transient.

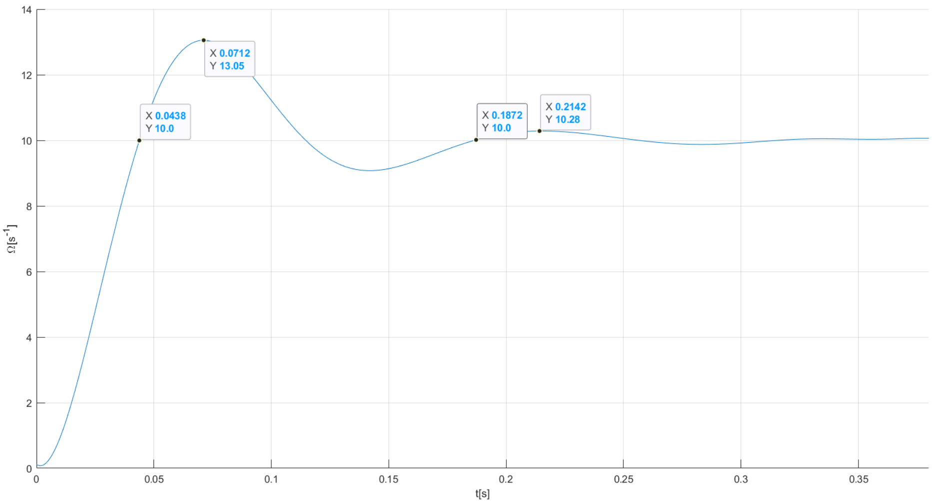

Figure 9: Smoothed input transient.

From the plot above the following values can be extracted:

- $\Omega_0 \approx 10.0[\frac{1}{s}]$
- $\Omega_{m1} = 3.05[\frac{1}{s}]$
- $\Omega_{m2} = 0.28[\frac{1}{s}]$
- $T_1 = 0.1434[s]$

Given the formulas from the lecture and laboratory description, the times constants can be calculated:

$$\omega_p = \frac{2\pi}{T_1} \approx 43.8158 \left[\frac{1}{s}\right] \tag{29}$$

$$\alpha = \frac{1}{T_1} ln\left(\frac{\omega_{m1}}{\omega_{m2}}\right) = 16.6535 \left[\frac{1}{s}\right] \tag{30}$$

$$\omega_n = \sqrt{\omega_p^2 + \alpha^2} = 46.8739 \left[\frac{1}{s}\right] \tag{31}$$

$$T_m = \frac{2\alpha}{\omega_n^2} = 0.0152[s] \tag{32}$$

$$T_a = \frac{1}{2\alpha} = 0.0300[s] \tag{33}$$

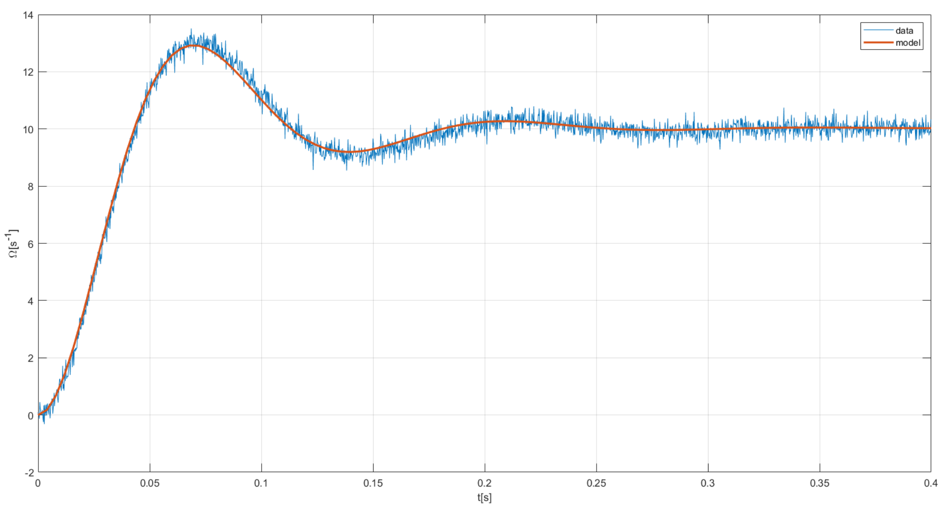

Figure 10: Model comparison with the data points.

So finally, the parameters are as follows:

- $T_a = 0.0300[s]$,
- $T_m = 0.0152[s]$.

3.2 Continuousness of the armature current

While deriving the formulas for the parameters T_a and T_m, some tools, which require the continuity of work, were used (e.g. integrals). The main idea behind this method is that the armature current of a motor powered from a thyristor rectifier is continuous, so if it wasn't, then another deriviation would have to be performed (as the previous one would not be relevant).