ANALYSIS AND SYNTHESIS OF CONTINUOUS SPEED CONTROL SYSTEM FOR DC ELECTRIC MOTOR DRIVES
Control System Elements course

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1 Task description

The goal of the exercise was to perform a synthesis of a continuous speed control system for a DC electric motor drive,examine the behavior of the system and adjust the parameters of the controller in accordance with the given quality indicators of the transient process.
The following block diagram is given:

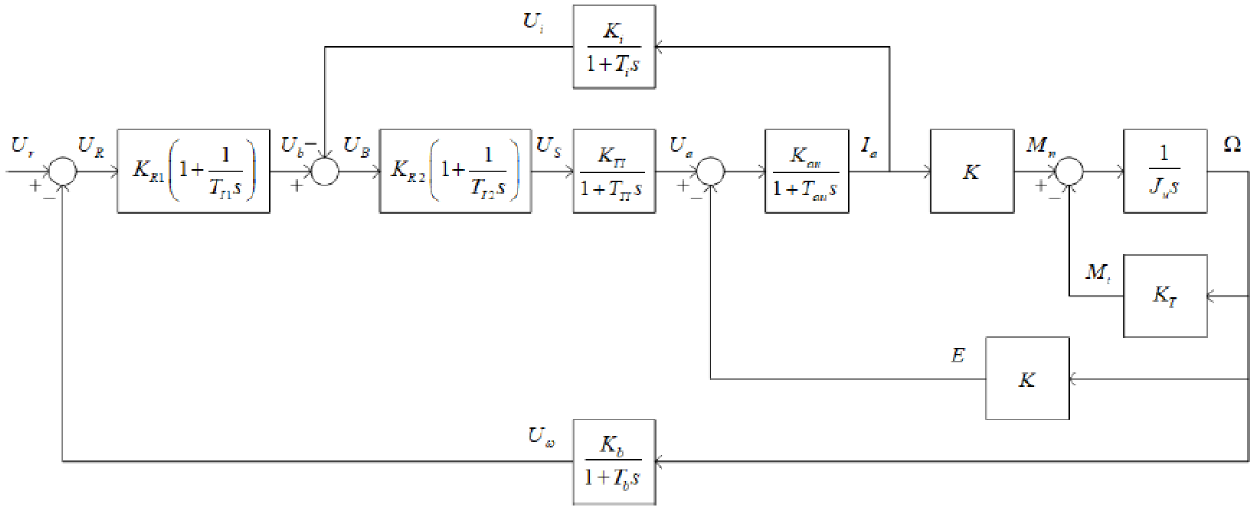

Figure 1: Block diagram of the speed control system of a DC electric motor drive loaded with a DC generator.

Parameters: $U_{an} = 220V$, $I_{an} = 3.4A$, $n_n = 1500r/min$, $I_{a0} = 0.3A$, $K_{TI} = 45V/V$, $T_{TI} = 0.005s$, $K_{au} = 0.0612A/V$, $T_{au} = 0.0184s$, $K = 1.211Vs$, $K_T = 0.01957Nms$, $J_u = 0.0157kgm^2$, $T_M = 0.8s$, $K_i = 1.57V/A$, $T_i = 0.005s$, $K_b = 0.065Vs$, $T_b = 0.025s$.

2 Tasks for preparation and testing the knowledge

2.1 Technical optimum criterion - armature current control

We have to determine the gain coefficient K_{R2} and time constant T_{I2} using Bode's display of frequency characteristics of the open loop of the armature current to achieve a given overshoot in the armature current response: $\sigma_m \in \{5, 10, 15, 20, 25, 30\}[\%]$. The feedback of the anti-electromotive force can be ignored.
The first step is to determine time constant T_{I2}. It should have the value of the maximum time constant from T_{TI}, T_i and T_{au}. In this case $T_{I2} = T_{au} = 0.0184s$. Next, the transfer function of the open loop system needs to be calculated:

$$G_o(s) = K_{R2}\left(1 + \frac{1}{T_{I2}s}\right)\frac{K_{TI}}{1 + T_{TI}s}\frac{K_{au}}{1 + T_{au}s}\frac{K_i}{1 + T_i s} \quad (1)$$

$$G_o(s) = \frac{K_{R2}K_{TI}K_{au}K_i(1+T_{I2}s)}{(1+T_{TI}s)(1+T_{au}s)(1+T_is)T_{I2}s} \tag{2}$$

$$G_o(s) = \frac{K_{R2}K_{TI}K_{au}K_i}{(1+T_{TI}s)(1+T_is)T_{I2}s} \tag{3}$$

Bode diagram is plotted for the system.

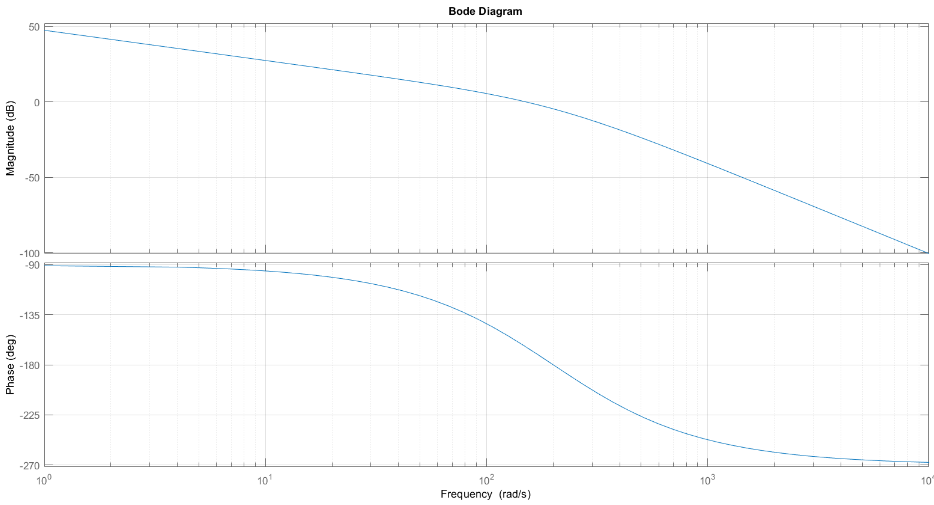

Figure 2: Bode diagram of the open-loop system.

With the formula $\gamma \approx 70^o - \sigma_m[\%]$, for each value of σ_m phase margin calculated. Then we calculate the angle γ_0, for which this phase margin occurs and read the corresponding ω. For this ω the corresponding value of the amplitude has to be found and K_R calculated on this basis, changed to linear scale and (if needed) changed a bit to obtained the proper overshoot.

$\sigma_m[\%]$	$\gamma[deg]$	γ_0 [deg]	ω [rad/s]	K_R [dB]	K_R	K_{R2}
5	65	-115	44.2	-14.1	0.197	0.21
10	60	-120	53.3	-12.3	0.243	0.2536
15	55	-125	62.9	-10.6	0.295	0.2953
20	50	-130	72.9	-9.07	0.352	0.3372
25	45	-135	82.4	-7.65	0.414	0.3801
30	40	-140	92.5	-6.39	0.479	0.4244

Table 1: Parameters for each overshoot value.

2.2 Approximate procedure from the frequency response of the open loop

The transfer function of the closed loop of the armature current is approximated in the following way:

$$G_{zi}(s) = \frac{K_{zi}}{1+T_{zi}s}, \tag{4}$$

where $K_{zi} = \frac{1}{K_i} \approx 0.6369$ and $T_{zi} \approx T_i + T_{TI} = 0.01$.

2.3 Technical optimum criterion - speed control

We have to determine the gain coefficient K_{R1} and time constant T_{I1} using Bode's display of frequency characteristics of the open loop of the speed to achieve a given overshoot in the speed response: $\sigma_m \in \{5, 10, 15, 20, 25, 30\}[\%]$.
The steps are as in the task with armature current control. First step is to determine time constant T_{I1}. It should have the value of the maximum time constant from all time constants in the control system. In this case $T_{I1} = T_M = 0.8s$.
Next, the transfer function of the open loop system needs to be calculated:

$$G_o(s) = K_{R1}\left(1 + \frac{1}{T_{I1}s}\right)\frac{K_{zi}}{1+T_{zi}s}\frac{K/K_T}{1+T_Ms}\frac{K_b}{1+T_bs} \tag{5}$$

$$G_o(s) = \frac{K_{R1}K_{zi}\frac{K}{K_T}K_b(1+T_{I1}s)}{(1+T_bs)(1+T_Ms)(1+T_{zi}s)T_{I1}s} \tag{6}$$

$$G_o(s) = \frac{K_{R1} K_{zi} \frac{K}{K_T} K_b}{(1 + T_b s)(1 + T_{zi} s) T_{I1} s} \tag{7}$$

Bode diagram is plotted for the system.

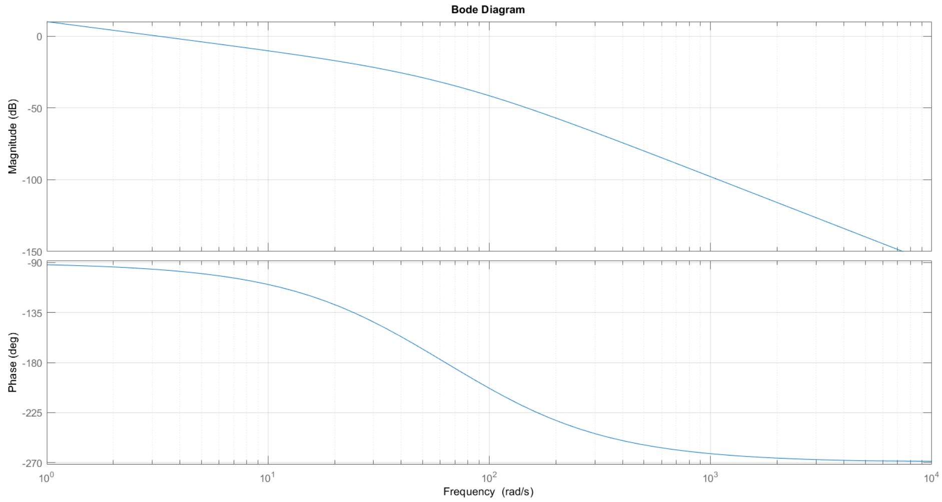

Figure 3: Bode diagram of the open-loop system.

With the formula $\gamma \approx 70^o - \sigma_m[\%]$, for each value of σ_m phase margin calculated. Then we calculate the angle γ_0, for which this phase margin occurs and read the corresponding ω. For this ω the corresponding value of the amplitude has to be found and K_R calculated on this basis, changed to linear scale and (if needed) changed a bit to obtained the proper overshoot.

$\sigma_m[\%]$	$\gamma[deg]$	γ_0 [deg]	ω [rad/s]	K_R [dB]	K_R	K_{R1}
5	65	-115	12.9	12.6	4.266	4.3
10	60	-120	15.6	14.5	5.309	5.17
15	55	-125	18.4	16.2	6.457	5.995
20	50	-130	21.4	17.8	7.782	6.81
25	45	-135	24.3	19.2	9.120	7.63
30	40	-140	27.6	20.7	10.839	8.47

Table 2: Parameters for each overshoot value.

2.4 Symmetric optimum criterion

We have to determine the parameters of the PI speed controller using Bode's display of the frequency characteristics of the open speed loop using symmetric optimum criteria, which obtain a symmetric amplitude-frequency characteristic in the vicinity of the cross-sectional frequency ω_{cs}. At the same time, it is necessary to achieve a given overshoot in the speed response: $\sigma_m \in \{5, 10, 15, 20, 25, 30\}[\%]$. First of all, the system has to be approximated with an astatic one. PT1 element with the biggest time constant is changed into an integral element (time constant doesn't change). Then ω_1 has to be calculated - it corresponds to the biggest time constant apart from the one used for making an integral element (in our case $\omega_1 = \frac{1}{T_b} = 40$). The following values need to be calculated for each overshoot value:

$$a = \frac{70 - \sigma_{\%}}{15}, \tag{8}$$

$$\omega_{c\omega} = \frac{\omega_1}{a}, \tag{9}$$

$$T_{I1} = \frac{a^2}{\omega_1}, \tag{10}$$

Therefore:

σ_m[%]	a	$\omega_{c\omega}$	T_{I1}
5	4.33	9.231	0.469
10	4	10	0.4
15	3.67	10.909	0.336
20	3.33	12	0.278
25	3	13.33	0.225
30	2.67	15	0.178

Table 3: Parameters for each overshoot value for symmetric optimum criterion.

The transfer function needs to be calculated:

$$G_o(s) = K_{R1}\left(1 + \frac{1}{T_{I1}s}\right)\frac{K_{zi}}{1 + T_{zi}s}\frac{K/K_T}{T_M s}\frac{K_b}{1 + T_b s} \tag{11}$$

$$G_o(s) = \frac{K_{R1}K_{zi}\frac{K}{K_T}K_b(1 + T_{I1}s)}{(1 + T_b s)(1 + T_{zi}s)T_M T_{I1}s^2} \tag{12}$$

It has to be checked that $\omega_{c\omega}$ is a crossover frequency, so for each overshoot value the corresponding T_{I1} and K_{R1} is used to plot a Bode diagram, then K_{R1} is read from the plot (for the proper value of $\omega_{c\omega}$.

σ_m[%]	$\omega_{c\omega}$	$I1$	K_R [dB]	K_R	K_{R1}
5	9.231	0.469	9.22	2.89	2.5
10	10	0.4	9.97	3.15	3.41
15	10.909	0.336	10.7	3.43	4.04
20	12	0.278	11.5	3.76	4.355
25	13.33	0.225	12.5	4.22	4.39
30	15	0.178	13.5	4.73	3.99

Table 4: Parameters for each overshoot value - symmetric criterion.

3 Work on the exercise

3.1 Armature current control test

The Simulink model of the armature current control system (without feedback of the anti-electric force) with the default change of input size $\Delta U_b = 10K_i$ (with the help of Matlab) was used to correct the values of the calculated controller parameters so that the set overshoots are achieved in the response.

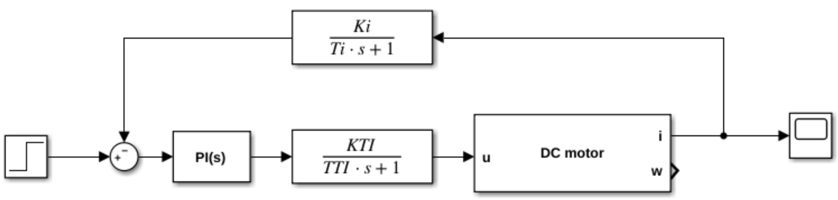

Figure 4: Simulink model for armature current control.

The overshoot was calculated in Matlab and then the gains were changed accordingly. Finally, the values presented below were obtained:

- σ_m = 5%: $K_{R2} = 0.2557$,
- σ_m = 10%: $K_{R2} = 0.2919$,
- σ_m = 15%: $K_{R2} = 0.3296$,
- σ_m = 20%: $K_{R2} = 0.3692$,
- σ_m = 25%: $K_{R2} = 0.4105$,
- σ_m = 30%: $K_{R2} = 0.4539$.

3.2 Speed control test

The Simulink model of the speed control system with the default change of input size $\Delta U_b = 10K_i$ (with the help of Matlab) was used to correct the values of the calculated controller parameters so that the set overshoots are achieved in the response.

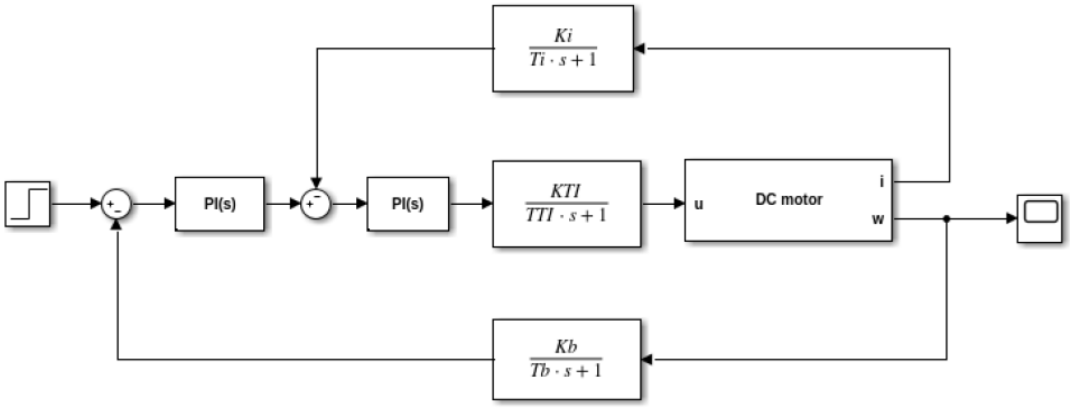

Figure 5: Simulink model for speed control.

The overshoot was calculated in Matlab and then the gains were changed accordingly. I assume that for calculating K_{R1} the overshoot guaranteed by K_{R2} would be 5%. Finally, the values presented below were obtained:

- $\sigma_m = 5\%$: $K_{R1} = 4.705$,
- $\sigma_m = 10\%$: $K_{R1} = 5.57$,
- $\sigma_m = 15\%$: $K_{R1} = 6.308$,
- $\sigma_m = 20\%$: $K_{R1} = 6.983$,
- $\sigma_m = 25\%$: $K_{R1} = 7.615$,
- $\sigma_m = 30\%$: $K_{R1} = 8.293$.